Space-Time Adaptive Processing: Algorithms

Wolfram Bürger
Research Institute for High-Frequency Physics and Radar Techniques (FHR)
Research Establishment for Applied Science (FGAN)
Neuenahrer Str. 20, D-53343 Wachtberg
GERMANY

buerger@fgan.de

ABSTRACT

In this lecture, we present some suboptimum STAP algorithms for radar, applied to moving target indication. In addition, we show some MTI results obtained with the multi-channel airborne experimental radar AER-II of FGAN-FHR.

1 INTRODUCTION

In the previous lecture [1], it was shown that if the interference situation (clutter, jamming, noise) is known, the optimum space-time adaptive filter vector $\mathbf{w}$ is given by

$$\mathbf{w} = \mu \mathbf{Q}^{-1} \mathbf{a}(\mathbf{u}_0, f_0), \tag{1}$$

where $\mathbf{a}(\mathbf{u}_0, f_0)$ denotes the given space-time signal vector,

$$\mathbf{Q} = E\left\{(\mathbf{c}+\mathbf{j}+\mathbf{n})(\mathbf{c}+\mathbf{j}+\mathbf{n})^H\right\} \tag{2}$$

is the space-time clutter-plus-jamming-plus-noise covariance matrix, and μ is a normalisation constant which can be chosen arbitrarily.

The large size of the space-time covariance matrix precludes the use of optimum space-time adaptive processing in an operational radar system, for it requires both an enormous computational effort and more training data than is usually available. Hence, the dimension of the filtering problem must be reduced. The resulting "suboptimum processors" will generally outperform optimum STAP if the covariance matrix is estimated from the data, since they require less computational effort and fewer training data.

2 ALGORITHMS

2.1 The Dimension of the Clutter Subspace

In the lecture on principles of adaptive array processing [5], subspace methods were introduced as a technique for reducing the amount of training data required for estimating the adaptive weights. This requires knowledge of the dimension $\dim CSS$ of the clutter subspace.

Bürger, W. (2006) Space-Time Adaptive Processing: Algorithms. In *Advanced Radar Signal and Data Processing* (pp. 7-1 – 7-12). Educational Notes RTO-EN-SET-086, Paper 7. Neuilly-sur-Seine, France: RTO. Available from: http://www.rto.nato.int/abstracts.asp.

For a side looking linear array, Brennan and Staudaher [6] found that

$$\dim CSS \approx \operatorname{int}\{N + (M-1)\gamma\},^{1} \tag{3}$$

where N is the number of array channels, M is the number of pulses, and

$$\gamma = \frac{2v_P T}{d} \tag{4}$$

is the ratio between the normalised clutter Doppler frequency $f_D T$ and the spatial frequency $\frac{d}{\lambda}\cos\beta$, with v_P the platform velocity, T the pulse repetition interval, and d the inter-element distance.

In general, equation (3), which is also known as Brennan's rule, only gives a lower bound on the number of clutter eigenvalues. It does not hold for forward looking planar arrays, as is illustrated by Figure 1, which shows the eigenvalues of an estimated space-time clutter-plus-noise covariance matrix, normalized to the noise power.

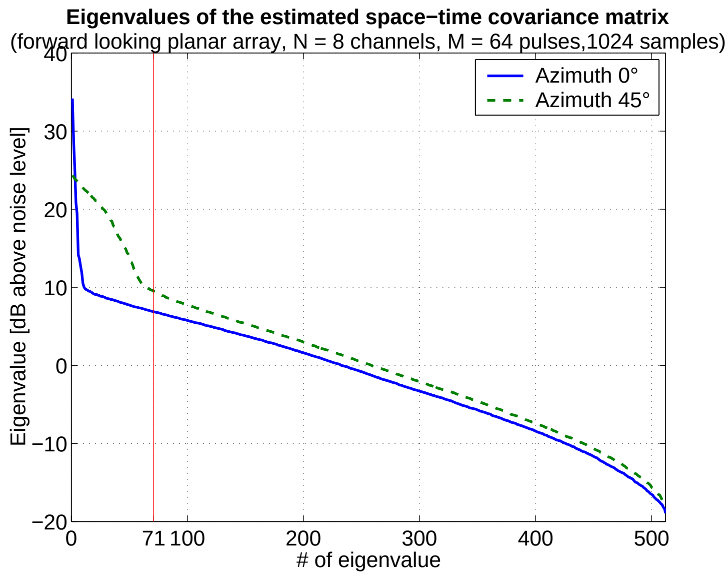

Figure 1: Eigenvalues of the Space-Time Covariance Matrix for a Forward Looking Planar Array.

2.2 The Displaced Phase Centre Antenna (DPCA) Technique

Historically, the displaced phase centre antenna (DPCA) technique was the first – and simplest – approach to space-time processing of clutter echoes. It was developed for implementation in RF technology, long time before the advent of digital signal processing. As illustrated in Figure 2, DPCA is based on a side looking antenna arrangement with two (or more) phase centres displaced along the flight axis. The aircraft motion can then be compensated by choosing the pulse repetition frequency (PRF) so that the second phase centre assumes the position of the first phase centre after one pulse repetition interval (PRI). Therefore, any two successive pulses received by the two different antenna parts appear to come from one phase centre fixed in space. A simple two-pulse canceller subtracts the second echo received by the trailing antenna from the first echo received by the leading antenna.

[1] A similar expression was found empirically by Klemm [7] almost ten years earlier.

It should be noted that the DPCA technique, being a deterministic space-time processing technique, requires identical channels, antenna phase centres displaced exactly along the flight axis, and a PRF perfectly matched to the platform velocity in order to work properly. However, due e.g. to channel errors and crab angle, these requirements will hardly be met in practice. Adaptive techniques have been devised to overcome this problem.

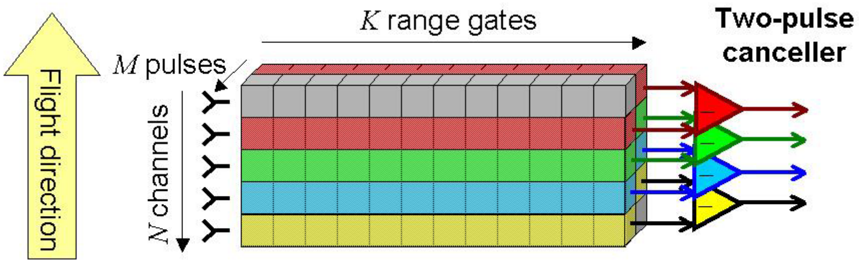

Figure 2: The Displaced Phase Centre Antenna (DPCA) Technique.

2.3 Post-Doppler STAP

Post-Doppler space-time adaptive processing techniques operate in the range-Doppler domain, i.e. after Doppler filtering has first been applied. The primary rationale for this is the dependence of the main beam clutter Doppler frequency on the cone angle (cf. [1]), which causes the clutter echoes to occupy only a few Doppler bins (while being present in all pulses), as is illustrated in Figure 3. This Doppler subbanding allows to use smaller adaptive filters, which require less computational effort and fewer training data. In addition, post-Doppler methods are less sensitive to element errors.

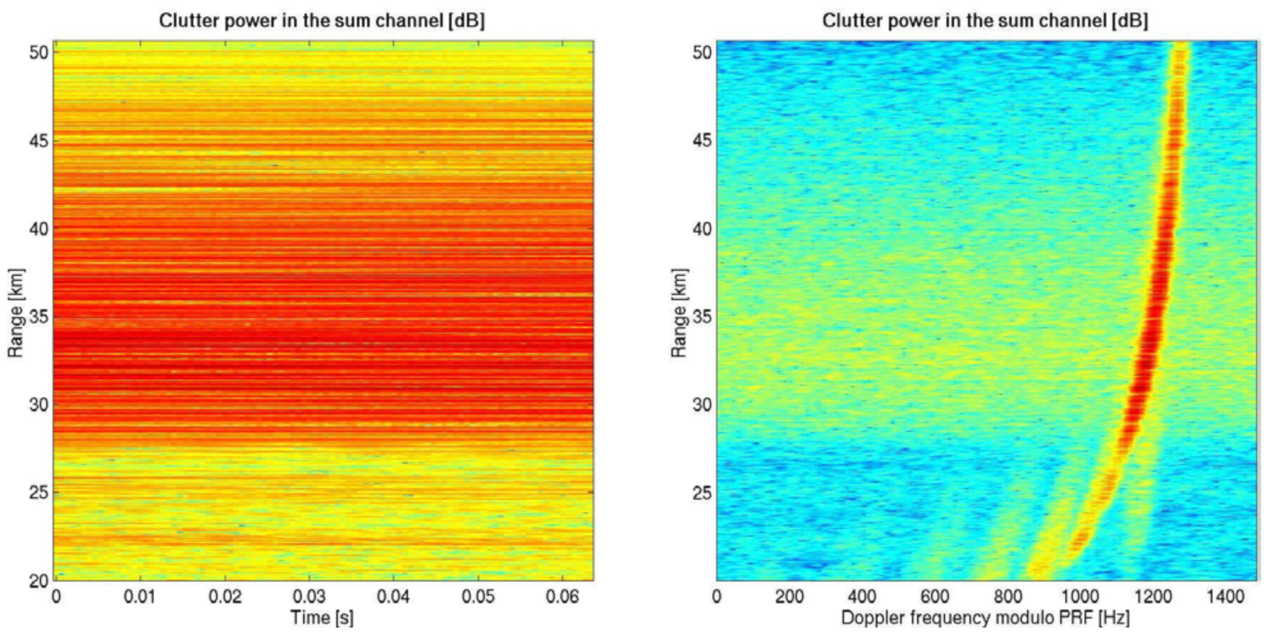

Figure 3: Clutter Power [dB] in the Sum Channel of a Forward Looking Radar, Plotted Over Range and Pulse-to-Pulse Time Resp. Doppler Frequency Modulo PRF.

Post-Doppler Adaptive Beamforming

The simplest post-Doppler STAP technique is *post-Doppler adaptive beamforming*, which has also been called frequency dependent spatial processing, or factored post-Doppler. In this method, the NM-dimensional space-time filtering problem is divided into M separate N-dimensional adaptive beamforming problems, as shown in Figure 4. Both the computational effort and the amount of training data required are thus reduced considerably.

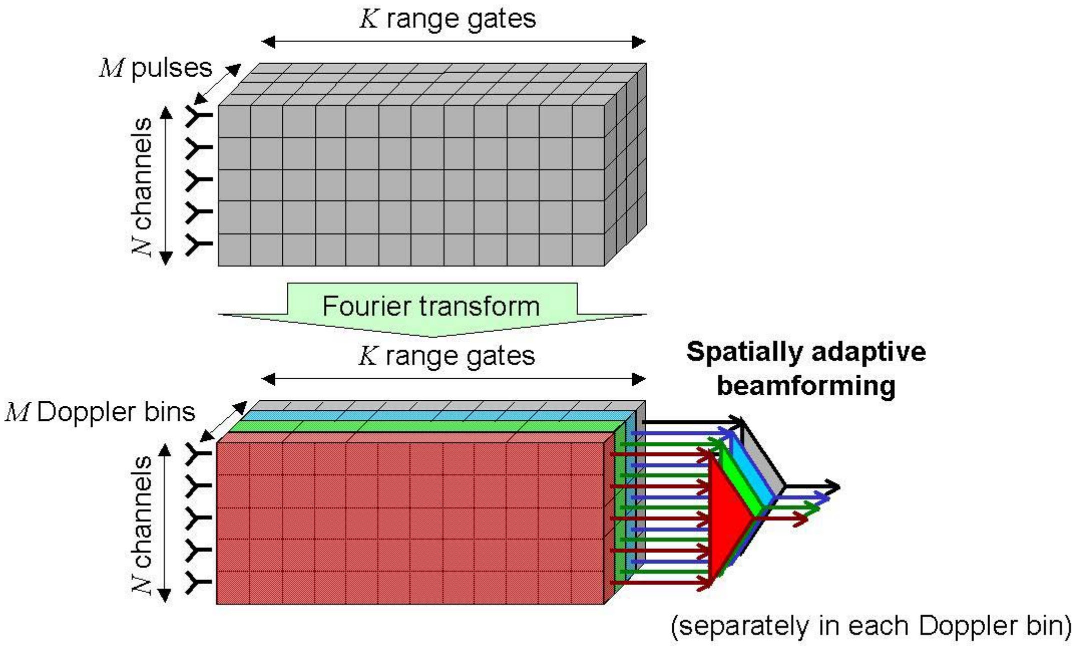

Figure 4: Post-Doppler Adaptive Beamforming.

Since post-Doppler adaptive beamforming is only spatially adaptive, the Doppler bins must be decorrelated in order for this technique to work well. As the correlation between Doppler bins is due to mainbeam clutter leakage into the Doppler sidelobes, it can be reduced by using a windowed Fourier transform, which leads to low Doppler sidelobes. However, the heavy tapering required also causes a broadening of the Doppler mainlobe and significant taper loss.

Post-Doppler adaptive beamforming is suitable for moving target detection using a multi-channel synthetic aperture radar, for the long pulse trains transmitted lead to high Doppler resolution.

Adjacent-Bin Post-Doppler STAP

The poor performance of post-Doppler adaptive beamforming can be improved by providing the architecture with temporal adaptivity. One way this can be achieved is by adaptively processing the spatial returns from several adjacent Doppler bins (*adjacent-bin post-Doppler STAP* [8]). In this method, which is also called extended factored STAP, the NM-dimensional space-time filtering problem is divided into M separate NL-dimensional space-time filtering problems, as shown in Figure 5. Both the computational effort and the amount of training data required are thus reduced considerably.

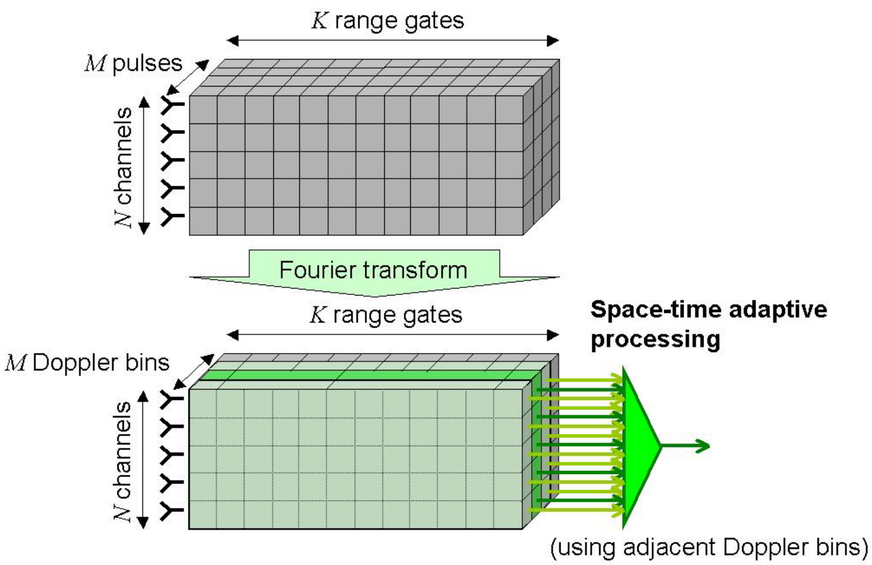

Figure 5: Adjacent-Bin Post-Doppler STAP.

Adjacent-bin post-Doppler STAP, while performing much better than post-Doppler adaptive beamforming, is sensitive to Doppler tapering, which increases the dimension of the clutter subspace [1].

PRI-Staggered Post-Doppler STAP

PRI-staggered post-Doppler STAP [6],[9] uses a bank of M'-pulse Doppler filters that produces $L = M - M'+1$ output pulses for each filter, see Figure 6. The spatial returns from the L output pulses are then processed adaptively. As in adjacent-bin post-Doppler STAP, the NM-dimensional space-time filtering problem is again divided into M separate NL-dimensional space-time filtering problems.

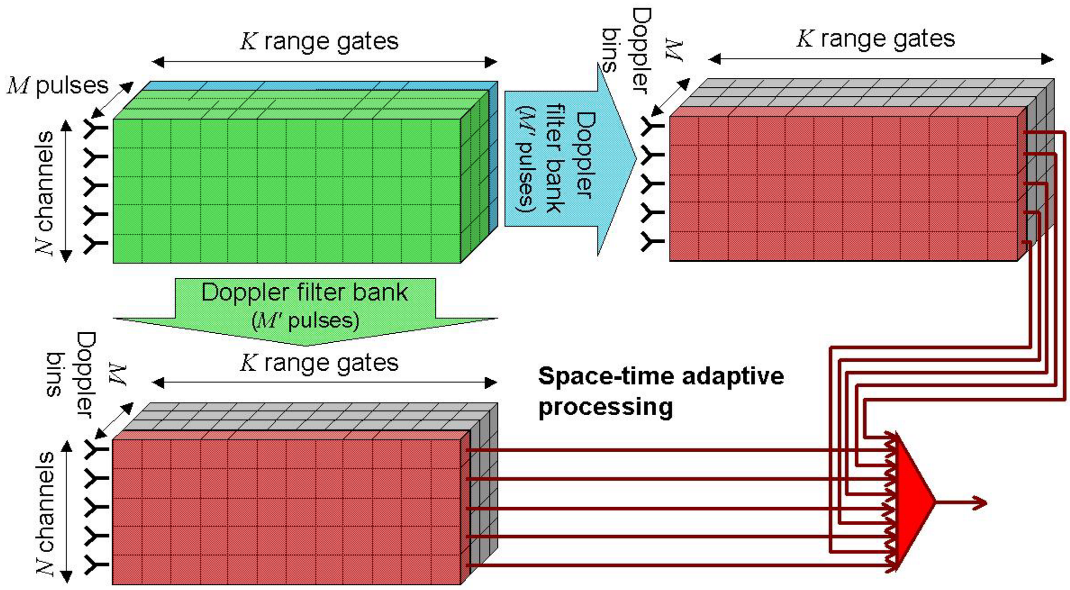

Figure 6: PRI-Staggered Post-Doppler STAP.

Given the same number of degrees of freedom and Doppler sidelobes, the performance of PRI-staggered post-Doppler STAP is usually better than that of adjacent-bin post-Doppler STAP [1].

Symmetric Auxiliary Sensor / Echo Processor

A space-time adaptive processor closely related to PRI-staggered post-Doppler STAP, and particularly well-suited for linear arrays, is described in [10]. It is called *symmetric auxiliary sensor / echo processor* and makes use of the fact that the concepts of overlapping uniform subarrays and symmetric auxiliary sensors, which are illustrated in Figure 7, lead to the same output but different processing schemes. In fact, the only difference between the two concepts is that all those central elements common to both subarrays are pre-summed. Analogously, the concepts of overlapping time intervals (which is used in PRI-staggered post-Doppler STAP) and symmetric auxiliary echoes lead to the same results but different processing schemes (in this case, the sum beamformers in Figure 7 must be viewed as Doppler filters).

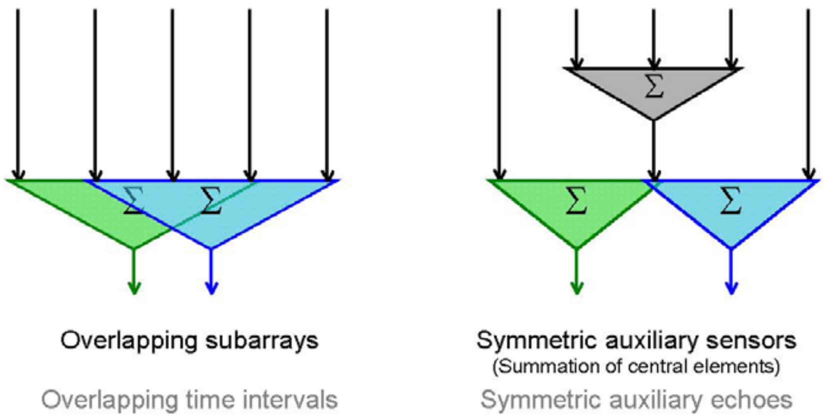

Figure 7: From Overlapping Subarrays to Auxiliary Sensors.

As shown in Figure 8, the *symmetric auxiliary sensor / echo processor* [10] uses both a central beamformer and a central Doppler filter, along with auxiliary sensors and auxiliary echoes, thus reducing the dimension of the adaptive problem – and both the computational effort and the amount of training data required.

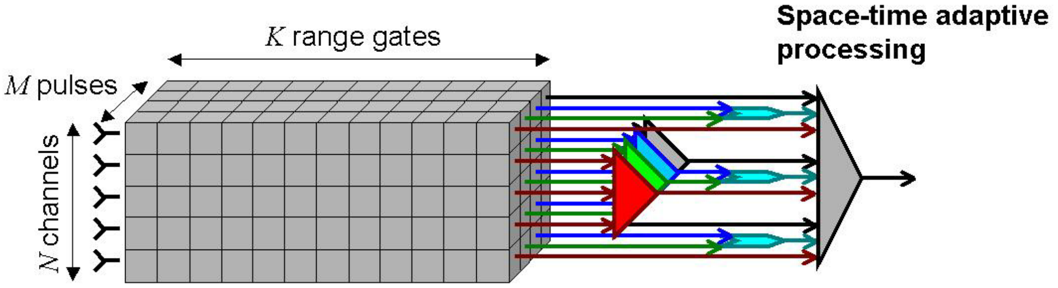

Figure 8: The Symmetric Auxiliary Sensor / Echo Processor (ASEP).

For linear arrays, the structure of the symmetric auxiliary sensor / echo processor leads to an efficient algorithm for determining the inverse of the space-time clutter-plus-noise covariance matrix.

Joint Domain Localized

As discussed at the beginning of this section, the rationale behind post-Doppler STAP is the fact that in view of the dependence of the main beam clutter Doppler frequency on the cone angle, clutter suppression in the range-Doppler domain is less complex than in the range-(pulse) time domain. In fact, this Doppler subbanding allows to use smaller adaptive filters, which require less computational effort and fewer training data.

Similarly, for array antennas with a large number of spatial channels, using beams formed from the outputs of these channels may be more economical. Such techniques are called beamspace STAP.

Joint domain localized (JDL, [11]) is probably the best-known beamspace post-Doppler STAP technique. In this method, the NM-dimensional space-time filtering problem is divided into $N'M$ separate $L'L$-dimensional space-time filtering problems, as shown in Figure 5. Both the computational effort and the amount of training data required are thus reduced considerably.

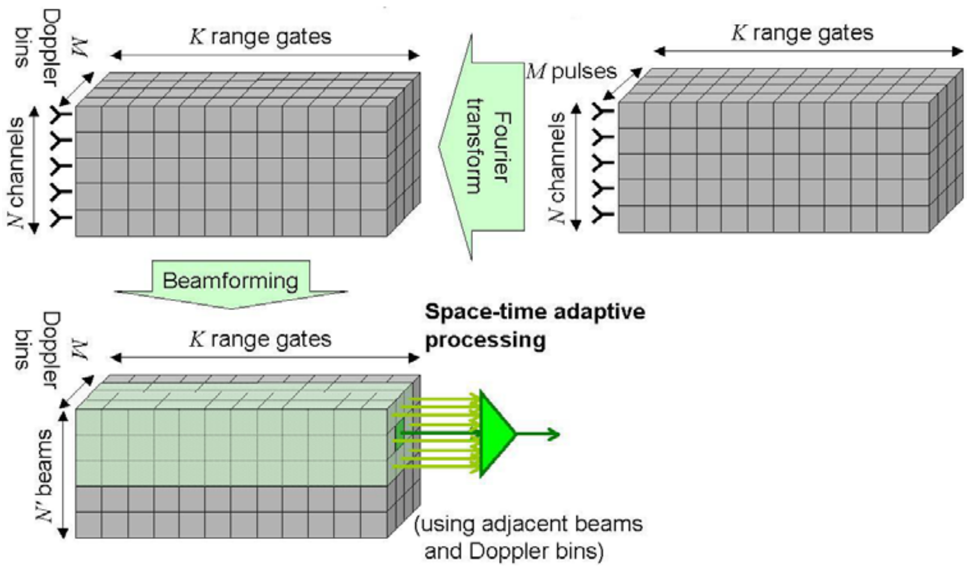

Figure 9: Joint Domain Localized (JDL).

2.4 Data-Dependent STAP

The rationale behind data-dependent STAP is the idea that adaptively reducing the dimension of the filtering problems will lead to a more efficient use of the available degrees of freedom. We will discuss these techniques only very briefly, as they are not specific of STAP. For details, the reader is referred to [3].

Principal Components Inverse (PCI)

Principal components inverse (PCI) and its modifications are subspace methods which replace the inverse of the space-time clutter-plus-noise covariance matrix by a weighted projection. It can be interpreted as a lean matrix inversion (LMI) method [5].

Multistage Wiener Filter

The *multistage Wiener filter* (MWF, [12]) selects a basis of the clutter subspace by successively choosing rank-one vectors so that the mean square error is minimized, but no knowledge of the full space-time covariance matrix is required.

2.5 The Space-Time Adaptive FIR Filter

Using the Cholesky decomposition

$$\mathbf{Q}^{-1} = \mathbf{R}^H \mathbf{R} \tag{5}$$

of the space-time clutter-plus-noise covariance matrix, the output of the optimum adaptive space-time filter can be written as

$$\begin{aligned} \mathbf{w}^H \mathbf{z} &= \left(\mu \cdot \mathbf{Q}^{-1} \mathbf{a}(\mathbf{u}_0, f_0)\right)^H \mathbf{z} \\ &= \mu^* \cdot \mathbf{a}^H(\mathbf{u}_0, f_0) \mathbf{R}^H \mathbf{R} \mathbf{z} \\ &= \mu^* \cdot \left(\mathbf{R}\mathbf{a}(\mathbf{u}_0, f_0)\right)^H \mathbf{R}\mathbf{z}. \end{aligned} \tag{6}$$

The triangular matrix $\mathbf{R}$ decorrelates (*whitens*) the received signal and matches the space-time steering vector $\mathbf{a}(\mathbf{u}_0, f_0)$. This whitening resp. matching can also be effected using a finite impulse response (FIR) filter.

The space-time adaptive FIR filter [13] is an extension of the linear prediction error filter to space-time signals. This filter can be used for minimizing the interference power in the received signal because it minimizes the prediction error in the least squares sense.

The coefficients of a space-time FIR filter operating in the temporal dimension are given by the first block column of the inverse of the covariance matrix $\mathbf{Q}_L$ of the space-time data segment of temporal length L. By including the spatial matching (secondary spatial beamforming) in the filtering operation, one obtains a filter vector of length NL, where N is the number of spatial channels.[2]

The space-time FIR filter offers several advantages:

- As the temporal length L of the filter can be chosen independently of the system dimensions (number of channels N, number of pulses M), the number of filter coefficients can be small, so that few training data are required, and the filtering operation can be carried out in real-time.
- The filter can also be used with inhomogeneous clutter or with a manoeuvring platform; however, it will then have to be re-adapted from pulse to pulse.

On the downside, the filtering operation causes some signal loss, as it shortens the data sequence from M pulses to $M-L+1$ pulses. This effect is visible in Figure 10, which shows the performance of the space-time adaptive FIR filter (*dashed line*) for a forward looking planar array antenna. Compared to optimum STAP using the subarray outputs (*solid line*), the space-time adaptive FIR filter shows some losses in pass band, but achieves the same narrow clutter notch. Conventional processing, i.e. beamforming plus Doppler filtering (*dotted line*), shows significant losses in pass band and does not achieve a narrow clutter notch.

[2] For details, the reader is referred to [2].

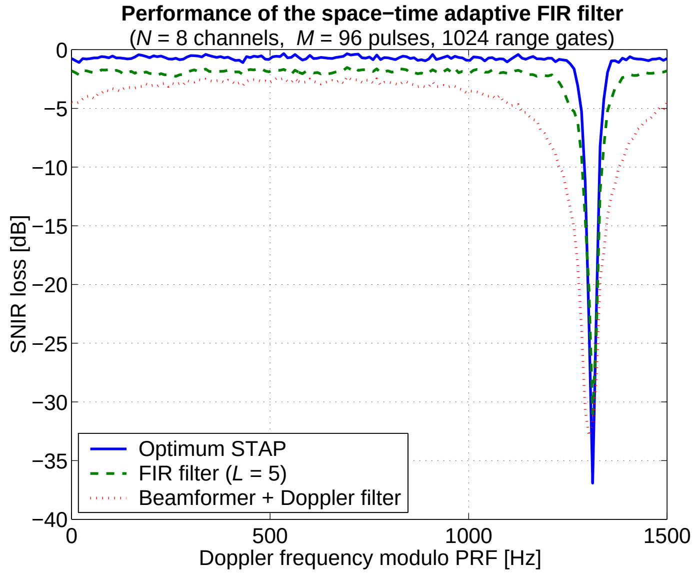

Figure 10: The Performance of the Space-Time Adaptive FIR filter (*solid line*) Compared to Optimum STAP Using the Subarray Outputs (*dashed line*) and Beamforming + Doppler Filtering (*dotted line*).

3 EXPERIMENTAL RESULTS

The results shown in this chapter were obtained using data acquired with the *Airborne Experimental Radar* (AER-II) of FGAN-FHR. This quasi-operational multi-channel X-band SAR/MTI radar system features an active electronically scanned array antenna. It has now been succeeded by the *Phased Array Multifunctional Imaging Radar* (PAMIR).

Figure 11 shows the outcome of a controlled SAR/MTI experiment with eight slowly moving ground targets. In both subplots, the signal power (in dB) is plotted over the ground coordinates. While the targets are hidden by clutter in the sum beam (left subplot), they are clearly visible after clutter suppression using post-Doppler adaptive beamforming (right subplot).

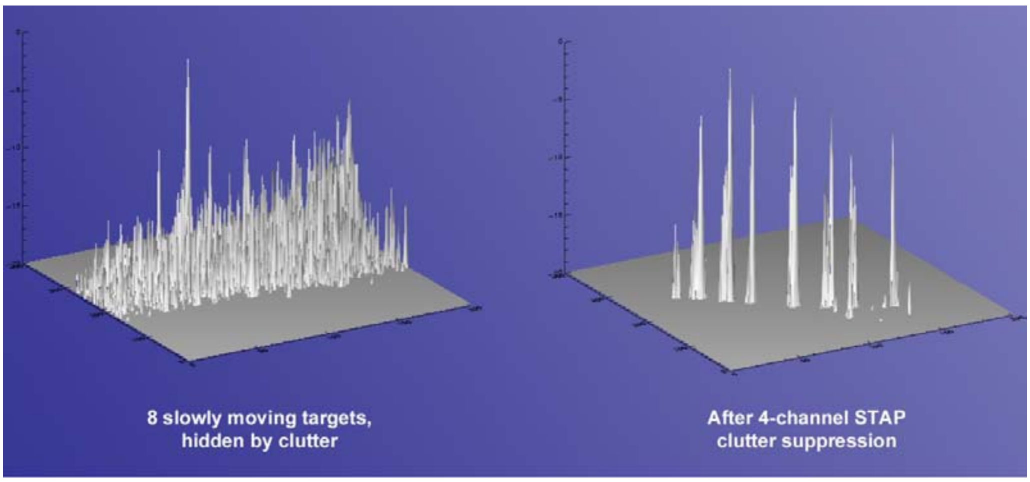

Figure 11: The Result of Space-Time Adaptive Processing: Signal Power [dB] Plotted Over (x, y) (*Image courtesy J. Ender*).

Figure 12 shows the result of combining a SAR image of the scene, in which the moving targets are not visible, with the GMTI information shown in the previous image.

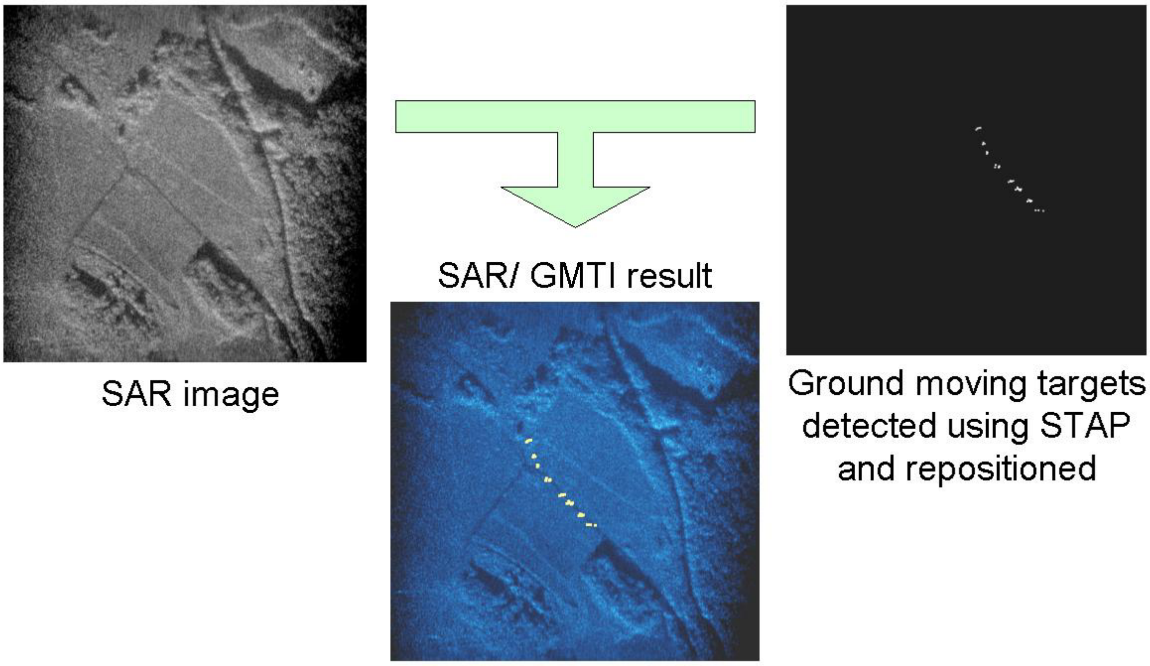

Figure 12: Fusion of SAR Image and GMTI Information (*Image courtesy J. Ender*).

Figure 13 shows the results of another SAR/MTI experiment, using targets of opportunity. Cars on a motorway junction were detected and correctly positioned; for some of them, tracks could be established.

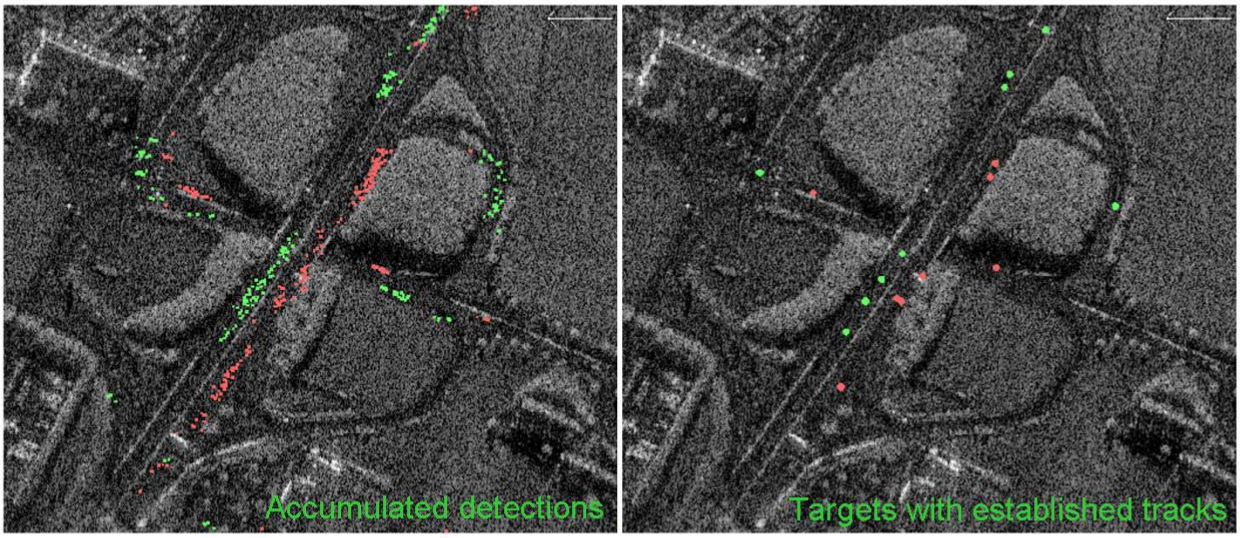

Figure 13: SAR Image of a Motorway Junction, Overlaid with the Accumulated Moving Target Detections (*left*) and those Targets for which Tracks could be Established (*right*) (*Image courtesy J. Ender*).

REFERENCES

[1] J. Ward, *Space-Time Adaptive Processing for Airborne Radar*, Technical Report 1015, MIT Lincoln Laboratory, Lexington, MA, USA, 1994 (available: http://handle.dtic.mil/100.2/ADA293032).

[2] R. Klemm, *Principles of Space-Time Adaptive Processing*, IEE, London, UK, 2002.

[3] J. Guerci, *Space-Time Adaptive Processing for Radar*, Artech House, Norwood, MA, USA, 2003.

[4] W. Bürger, *Space-Time Adaptive Processing: Fundamentals* (Paper No. 6 in this volume).

[5] U. Nickel, *Principles of Adaptive Array Processing* (Paper No. 5 in this volume).

[6] L. Brennan and F. Staudaher, *Subclutter visibility demonstration*, Technical Report RL-TR-92-21, Adaptive Sensors Incorporated, Santa Monica, CA, USA, 1992.

[7] R. Klemm, *Adaptive Clutter Suppression for Airborne Phased Array Radar*, IEE Proceedings F, Vol. 130, No. 1 (1983), pp. 125-132.

[8] R. DiPietro, *Extended Factored Space-Time Processing for Airborne Radar Systems*, in Proc. 26th Asilomar Conference on Signals, Systems, and Computers, Pacific Grove, CA, USA, 1992, pp. 425-430.

[9] J. Ward and A. Steinhardt, *Multiwindow Post-Doppler Space-Time Adaptive Processing*, in Proc. 7th IEEE Workshop on Statistical Signal and Array Processing, Quebec City, Canada, 1994, pp. 461-464.

[10] R. Klemm, *Space-Time Adaptive Processing: Principles and Applications*[3], IEE, London, UK, 1998.

[11] H. Wang and L. Cai, *On Adaptive Spatial-Temporal Processing for Airborne Surveillance Radar Systems*, IEEE Transactions on Aerospace and Electronic Systems, Vol. 30, No. 3 (1994), pp. 660-669.

[12] J. Goldstein, I. Reed and L. Scharf, A Multistage Representation of the Wiener Filter Based on Orthogonal Projections, IEEE Transactions on Information Theory, Vol. 44, No. 7 (1998), pp. 2943-2959.

[13] R. Klemm and J. Ender, *Multidimensional Filters for Moving Sensor Arrays*, in Proc. IASTED International Conference on Signal Processing and Digital Filtering, Lugano, Switzerland, 1990, pp. 9-12.

[14] J. Ender, *Space-Time Processing For Multichannel Synthetic Aperture Radar*, IEE Electronics and Communications Engineering Journal, Special Issue on STAP, Vol. 11, No. 1 (1999), pp. 29-38.

[3] This book is the "first edition" of [2].